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UNDERSTANDING THE DESIGN INTENT THROUGH THE ANALYSIS OF RENAISSANCE DRAWINGS
THE DIGITAL RECONSTRUCTION OF AN UNBUILT MAUSOLEUM BY GIULIANO DA SANGALLO

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Abstract

Giuliano da Sangallo is one of the paramount figure emerging from the Italian architecture during the Renaissance. He was commissioned to build many buildings, leaving also an ample documentary corpus, made of sketches and more technical drawings. On some pages in the Codice Barberiniano, a rich collection of drawings stored at the Vatican Apostolic Library, Giuliano drafted a plan and a section view of a building that recent studies speculate to be a conceptual proposal for the Pope Julius II's mausoleum. Beginning from these graphical representations, and taking into account many coeval paintings illustrating architectural details, several digital reconstructions were proposed and compared in two master thesis works, ending in a virtual computer model whose shape and proportions are expression of a plausible constructive hypothesis. Many analysis were also carried out on the model, in order to better understand the original Sangallo's design intent.

Keywords

Renaissance Drawings, Antonio da Sangallo, Digital 3D modeling, Architectural virtual reconstruction, Semantic BIM.

1. *Introduction*

Born in Florence in 1445 and there died in 1516, Giuliano da Sangallo was a multi-faceted architect, a sculptor, a woodworker and a military engineer.

Son of Francesco and brother of Antonio, Giuliano certainly gave his contribution to a family whose intense and creative activity led to many remarkable examples of Renaissance architecture.

From the elegance of Santa Maria delle Carceri central-plan church in Prato to the austerity expressed by the Lorenzo de' Medici's private villa in Poggio a Caiano, Giuliano da Sangallo could convey extraordinary complexity into his peculiar design process.

Initially working in Florence, then in Rome, Giuliano re-elaborated the classical legacy of previous ages, following the enthusiasm of his patrons for the architectural principles in the construction process. An impressive amount of sketches, drawings and surveys, collected mostly in the *Taccuino Senese* and in the *Codice Barberiniano*, shows the Giuliano's real passion for the ancient architecture throughout his whole professional career and life.

After being hired by Cardinal Giuliano della Rovere, who commissioned the Della Rovere palace in Savona, in 1496 Giuliano followed the prelate in Provence, where he freely deepened his knowledge on ancient monumental buildings. In 1504, following the election of Giuliano della Rovere to the papal throne with the name of Pope Julius II, Giuliano left for Rome, where he was longing for receiving new commissions from his former patron.

Many of his works, even unbuilt, are reported in many drawings by Giuliano da Sangallo himself, as he was one of the first Renaissance architects whose graphical corpus has been preserved by his heirs. It is still possible to examine today 21 paper sheets that are stored at the Gabinetto dei Disegni e delle Stampe degli Uffizi in Florence, together with the *Taccuino Senese*, an anthology of practical architecture now at the Biblioteca degli Intronati in Siena, and the *Codice Barberiniano* (Ms Barberini Lat. 4424) at the Vatican Apostolic Library. This last collection of classic architectural details such as capitals, mouldings, pediments and basements, shows on its frontispiece the year MCCCCLXV, the very beginning of Giuliano's experience in Rome.

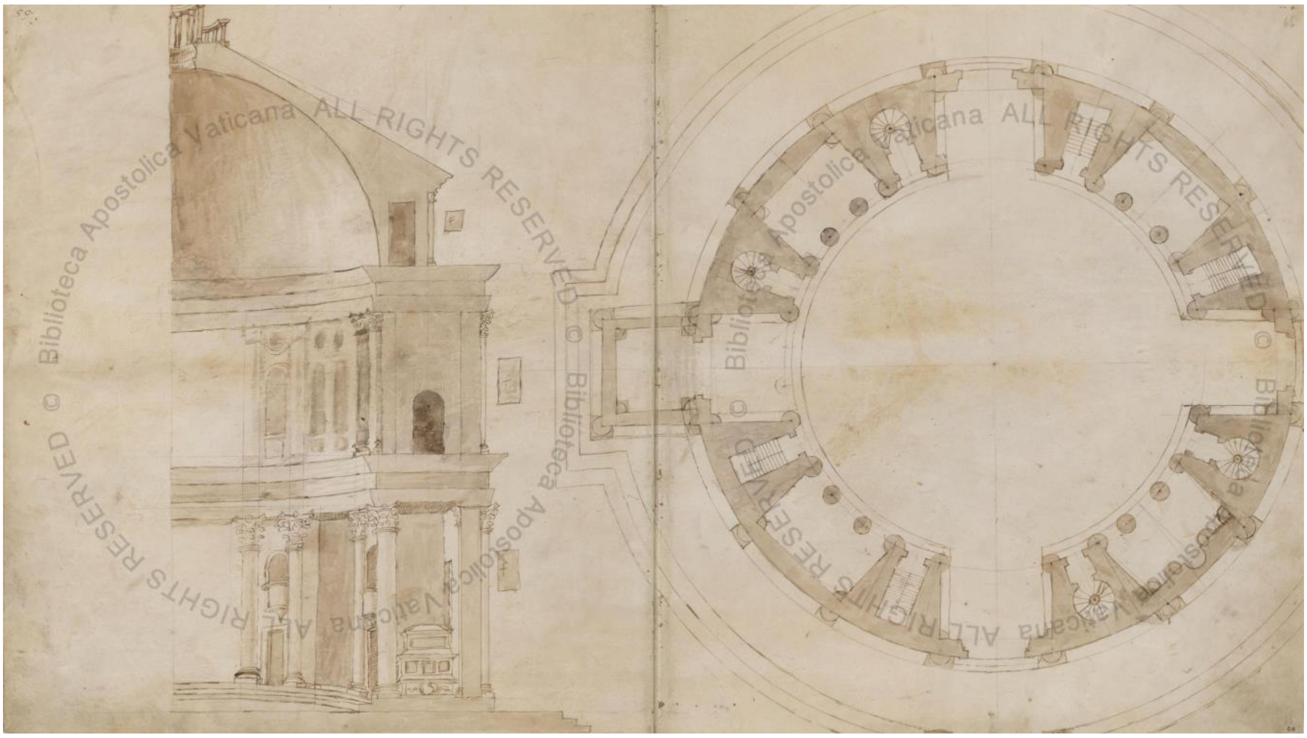

Fig. 1: The original drawings by Giuliano da Sangallo: the central-plan building investigated in this work, represented with a section view (51v) and a plan view (66r) in the Codice Barberiniano (Biblioteca Apostolica Vaticana).

In the *Codice Barberiniano*, Sangallo represented a section view (51v) and a plan view (66r) that are somehow related to each other (Figure 1): it is clearly visible part of the plan in the right side of the section view, as these sketches were probably conceived together.

These two drawings, which were identified with the proposal for Julius II's magnificent building destined to host his tomb (Frommel, 2014), were the starting data source for the digital reconstruction model presented in this work, which is the result of two master thesis in Architectural Engineering recently defended at the School of Architecture at the Alma Mater Studiorum - University of Bologna. Without going any further through the historical and architectural discussion, which is neither the research core nor the specific field of the authors, the graphical interpretations and the drawing hypothesis for a feasible visualization of the virtual reconstructed model will be introduced.

2. *The drawing proportions*

In the afore-mentioned section view, likely dated back to 1505, the overlapping Corinthian order and its heavy trabeation are only briefly outlined by the architect just like the lantern, as a reminiscence of the Brunelleschi's Sacrestia Vecchia. Sangallo added probably later strong watercolor shades, perhaps when he hastily copied the project into the *Codice Barberiniano* and when he ostensibly introduced the compartment for the sepulchral chapel.

Taking advantage of medium resolution images (Jpeg file format, 1781x2175 pixel wide), which can be freely downloaded from the Biblioteca Apostolica Vaticana web portal, the Sangallo's digitized drawings were translated into vector graphics, retracing all the drafted lines and circles in a CAD environment. Utmost attention was put on the lines retracing, performed mostly and carefully by hand following the central pixel for every intelligible line axis visible in highly magnified raster files imported into CAD software. A customized layer scheme was chosen in order to better evaluate proportional units, as it will be introduced later. The analysis of Sangallo's sketches plays a fundamental role, since the architect did not provide any scale representation, forcing to formulate several hypotheses about proportions that could have been actually deployed. The prevailing hypothesis (Frommel, 2014) privileges the adoption of a

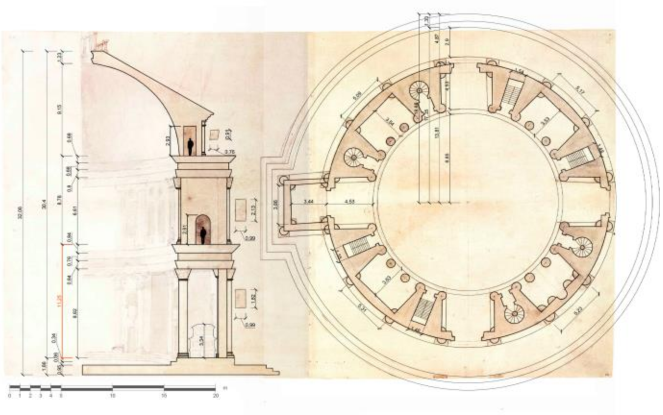

Fig. 2: The first reconstructive hypothesis with proportions superimposed to the original drawings.

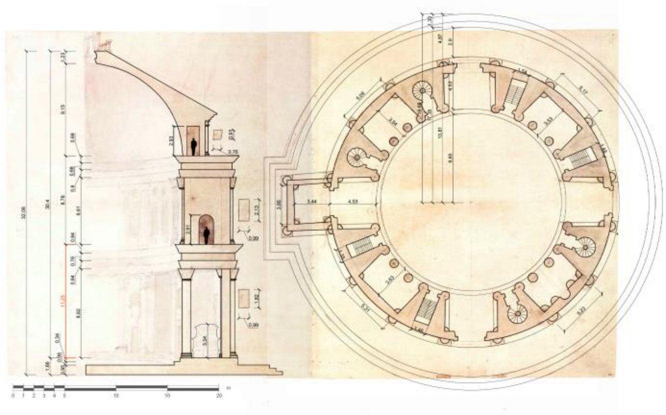

Fig. 3: The second reconstructive hypothesis with dimensions slightly higher than previous ones.

typical measurement unit in the Renaissance, the *braccio fiorentino*, a Tuscan module equal to 58 modern centimeters. Hereby follows a description of the afore-mentioned conjectures, with a focus on the reason why the *braccio forentino* application prevailed on others coeval references.

2.1 Hypothesis no. 1

Following an early review of the sketches, columns height was supposed to be 11.25 meters and the dome, stemming from a 20 meters height, would have been 18 meters in diameter. However, endorsing such hypothesis and scaling accordingly the whole drawing, both second and third order deambulatories would have been 2.9 meters high and such size is definitely out of proportion for the monument's functional purpose (Figure 2).

2.2 Hypothesis no. 2

In the Sangallo's plan view there are eight staircases: four are in spiral form, while four have two parallel rising stairs. After assuming one meter width per flight and scaling proportionally both plan and section views according to this assumption, it seems the second and third order deambulatories would reach an excessive height (Figure 3).

2.3 Hypothesis no. 3

Since the Corinthian order proportion, mostly used by Sangallo, ranges from a 1:10 to a 1:11 ratio, it was checked whether such order was effectively applied to all drawing's columns: the first floor only reached such proportion (1:10,5), while applying the Corinthian ratio to the others would have substantially altered the whole prospect (Figure 4).

2.4 Hypothesis no. 4

This hypothesis was tested by analysing a sketch drawn on the *Taccuino Senese* by Giuliano himself. Such drawing shows one column for each of the three different orders: Doric, Ionic and Corinthian. The last one is divided into 13 and 20 parts; this way, the mausoleum section view's columns were divided into such proportions in order to find a basic parametric module that the architect could have used as the drawing proportional module. The same speculation was applied to the sketched trabeation. Signs found on the drawing made unclear whether those architectural elements were divided into three or four parts, both divisions were applied on Sangallo's mausoleum, thus splitting at first the trabeation into three and later into four parts, in order to find the right module to be applied to the whole monumental section view.

As a matter of fact, neither the module based on the columns subdivision nor the one based on the trabeation were suitable for all of the building levels (Figure 5).

2.5 Hypothesis no. 5

The last interpretation was inferred by some studies already published on Santa Maria delle Carceri (Frommel, Gaiani & Garagnani, 2018).

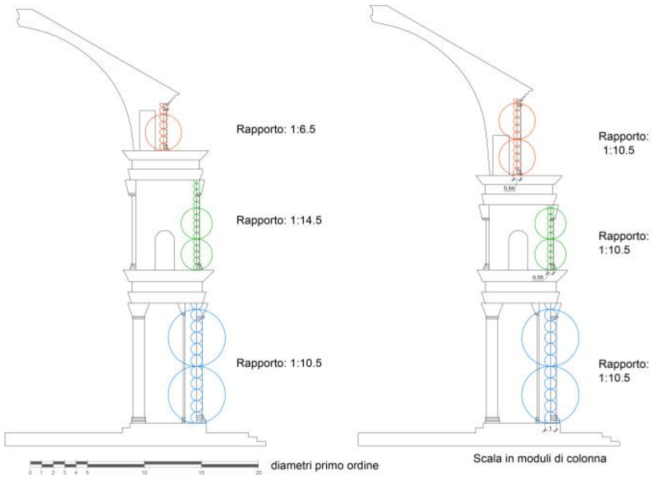

Fig. 4: The third reconstructive hypothesis with dimensions applied to the section view to determine the building height.

The central plan of such building shows a distinctive symbology (an original metric scale sketched by Giuliano and consisting in five small ink points) also found on mausoleum's plan view. Starting from the assumption that the distance between these points is a basic module equal to 1 *braccio fiorentino*, the distance between each architectural element becomes a round value and columns' diameter equals exactly to one module.

This has been considered as the most plausible hypothesis to scale the drawings; consequently, both plan and section views showed an appropriate representation scale according to the *braccio fiorentino* module.

Once this basic module had been chosen, the original drawings were fully proportioned using multiples and submultiples of the *braccio fiorentino* (Figure 6). Those multiples and submultiples are listed below:

- 1 braccio fiorentino = 58 cm.
- 1 passetto = 2 bracci = 116 cm.
- 1 soldo = 1/20 braccio = 2,9 cm.
- ½ braccio = 10 soldi = 29 cm.
- ¼ braccio = 5 soldi = 14,5 cm.
- ¾ braccio = 15 soldi = 43,5 cm.

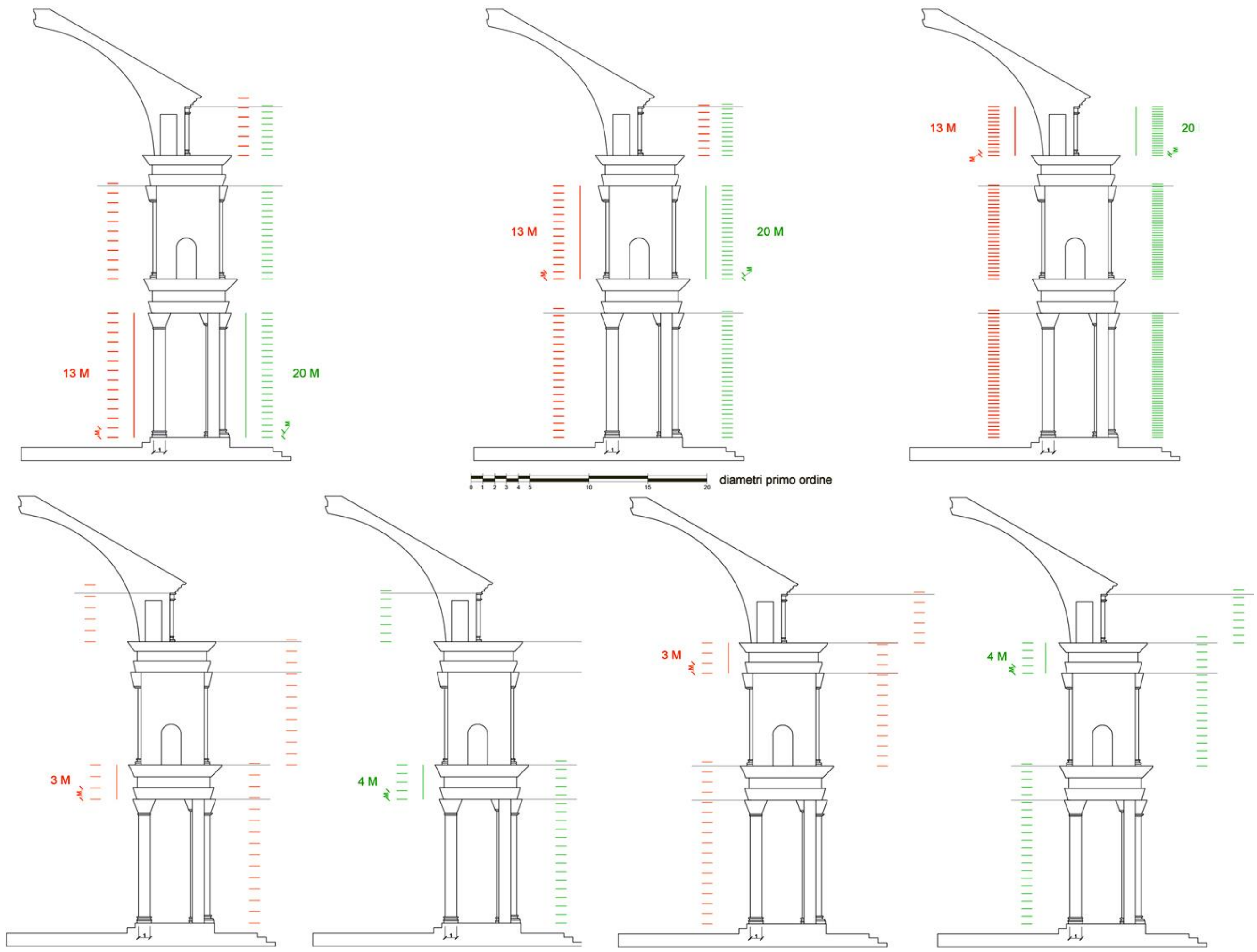

Fig. 5: The fourth reconstructive hypothesis with the Corinthian order divided into 13 and 20 parts to identify Sangallo's intent.

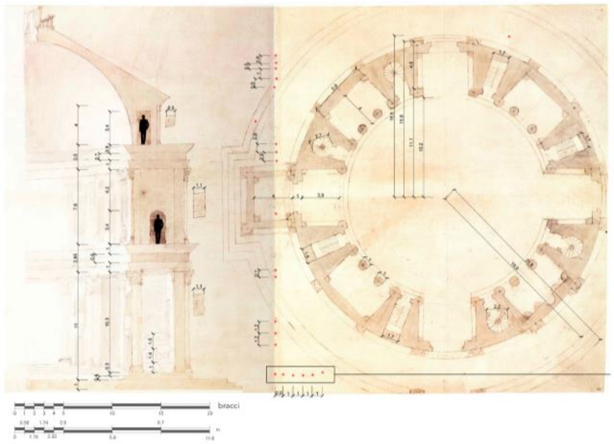

Fig. 6: The last and definitive reconstructive hypothesis with the *braccio fiorentino* as a metric proportional unit.

3. *Variations and hypothesis from the painted architecture*

Obviously, drawings in the *Codice Barberiniano* do not cover each and every architectural detail of the monumental building so many other works by Giuliano erected in Tuscany and Lazio were studied to explore many possible philological solutions to the several problems arising from the 3D modeling approach. The Santa Maria delle Carceri in Prato (1486-94) was studied as well as the Madonna dell'Umiltà sanctuary in Pistoia together with the Santo Spirito Sacristy (1489-92) and the Gondi Chapel in Santa Maria Novella, both in Florence, considering many elements inferred by scholars (Frommel and Wolf, 2016). Even paintings depicting similar buildings from the same period were considered, such as "*the Ideal City*" in Urbino dated to the late 15th century and the "*Consegna delle Chiavi*" by Perugino in the Sistine Chapel, among others. Also existing buildings like the Pantheon or Santa Costanza's mausoleum in Rome were inspiring as well as classical treatises by Vignola (Vignola, 1562), Serlio (Serlio, 1537) or Alberti (Alberti, 1443). When neither coeval monumental buildings nor paintings could help tridimensional reconstruction, evaluating hypothesis were developed according to a constructive point of view. In wider terms, a process of reconstruction is essentially composed by decisions based on various sets of assumptions that may be obvious to the scientific curator of the reconstruction process but not to the general public, the final user or those who could later see

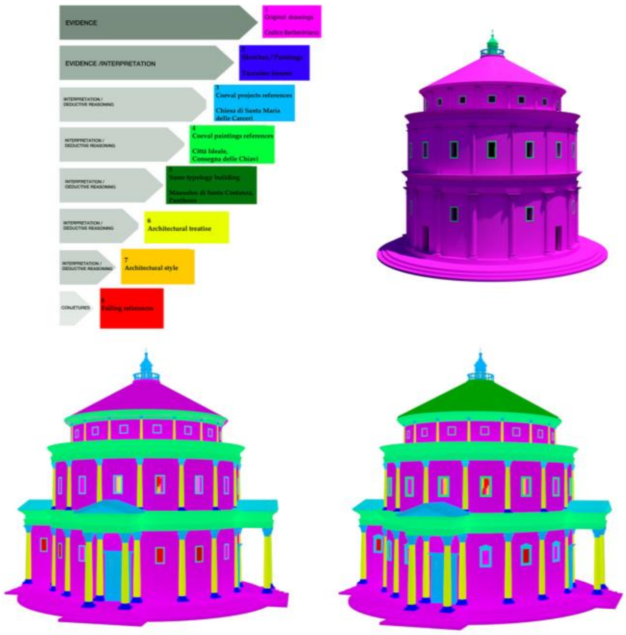

Fig. 7: The pseudo-color scheme applied to preliminary, intermediate and final stage, to identify levels of uncertainty.

the final restitution. This subjectivity, if not correctly reported, compromises the validity of an entire virtual reconstruction. Therefore, in order to validate the entire 3D modelling process and to ease the exchange and reuse of information between experts in various disciplines on the adopted conjectures, it is mandatory that the 3D geometric models are clearly demonstrative of the solutions adopted to meet the uncertainties and the frequent lack of information. Among all methodologies adopted and proposed for representing probability, ambiguity, reliability or uncertainty in 3D reconstructions, the use of color is undoubtedly the most efficient and unambiguous approach, because it allows a clear understanding according to widely shared semantic codes, portraying the degree of uncertainty surrounding the hypothetical reconstruction of each element of an artefact. Thus, the different sources of information used throughout mausoleum's process of digital reconstruction were organised into a hierarchy according to their reliability, producing a list characterised by a growing level of uncertainty:

- Sangallo's original drawings (*Codice Barberiniano*);
- Sangallo's sketches (*Taccuino Senese*);
- Sangallo's coeval projects (*Santa Maria delle Carceri, Santa Maria dell'Umiltà, Cappella Gondi*, etc…);

- Coeval paintings portraying the same kind of monumental buildings (*Ideal city, Consegna delle Chiavi*);
- Same typology buildings (*Santa Costanza mausoleum, Pantheon*);
- Architectural treatises (*Serlio, Alberti, Vignola*);
- Conjectures.

The initial classification provided is specific for Sangallo's mausoleum, relying on his sketches, while the last categories refer to the conjectural hypothetical reconstructions made, due to the lack of any documentary source or references, by using the *"common/scientific sense"* of the researchers based on their accumulated knowledge or, when necessary, on their imagination. Each category was therefore identified by a corresponding RGB-HSV color space.

This was applied to each architectural element of the reconstructed model, thus identifying its corresponding degree of uncertainty (Apollonio, 2016).

Therefore, the use of this method visually assesses the proper level of knowledge related to the reconstructive process; highlighting what is unknown rather than hiding it. The *"pseudo-color"* scheme was applied to mausoleum digital model both in a final stage and in a preliminary stage (Figure 7).

Images clearly show how the initial model is more reliable but also less detailed than the final one. This indicates that reliability of architectural elements is strictly related to level of detail: augmenting the latter means diminishing the former.

Modelling a more detailed building drawing from different sources is something legitimate if the level of reliability of each architectural element is shown, otherwise the validity of the entire reconstruction is compromised.

This way visualisations of the real and contemporary objects that are more interpretative than philological can be created.

In this case the reconstruction expresses a desire to explicitly describe and represent a whole story: not only a "*re-design*" to understand, but a "*re-interpretation*" to communicate (Figure 8).

4. *The reconstruction of a plausible 3D building*

All the bi-dimensional vector graphics prepared from the previous analysis were later used in order to retrace architectural components' shapes in a 3D modeling software.

Fig. 8: The development of uncertainty knowledge applied to semantic elements: as more information is inferred by different sources, the final 3D model improves.

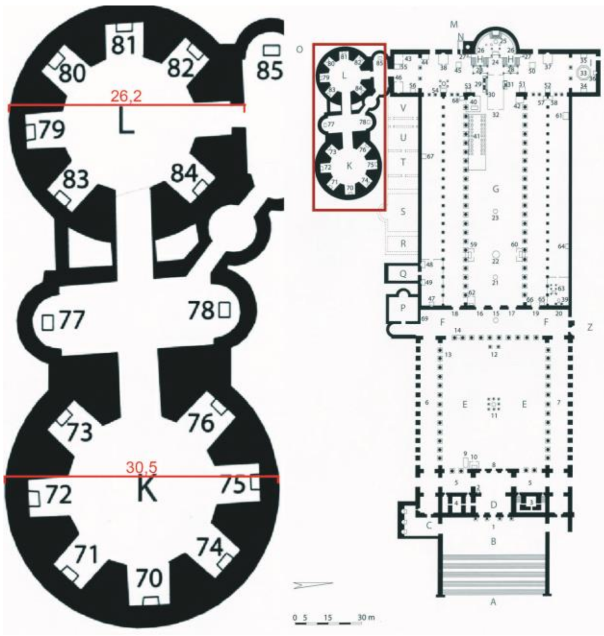

Fig. 9: The two circular buildings (*rotundas*) close to the Saint Peter's Basilica in Rome: Santa Petronilla and Saint Andrew.

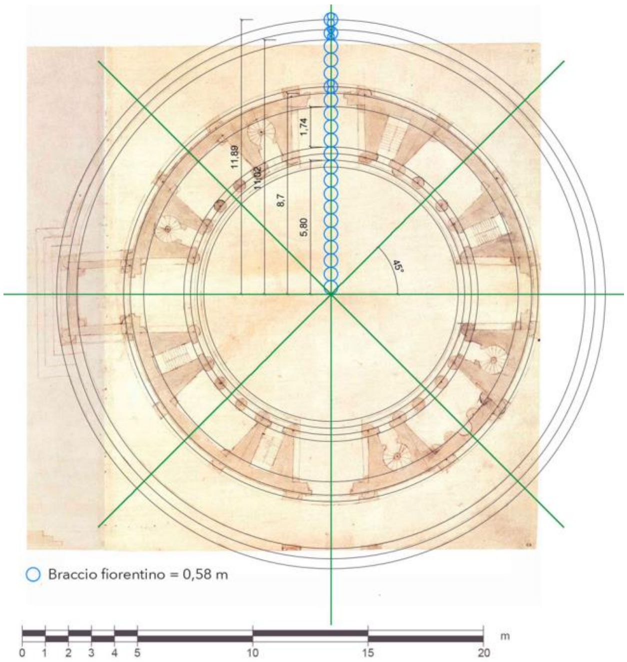

Fig. 10: Diagonals lines applied following a geometric symmetry, superimposing CAD elements to the plan view.

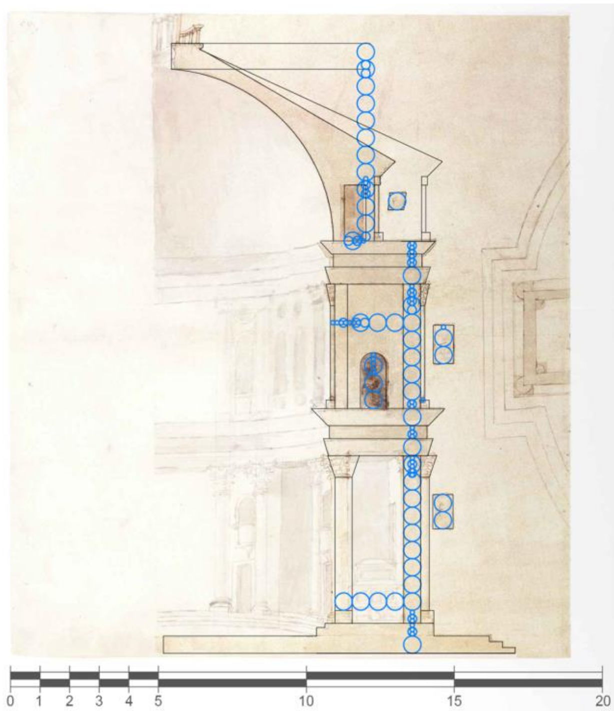

Fig. 11: The braccio fiorentino proportional unit recognized as the idel module for the final 3D modeling stage.

Thus, CAD drawings were imported into *Autodesk 3D Studio Max 2016*, keeping the original layering structure of the initial vector graphics. Since many hypothesis were studied for the development of a feasible 3D reconstruction, some incremental criteria were adopted (De Luca, 2014), increasing the semantics of components as the knowledge on the building was improving.

An initial box modeling stage was performed in order to replicate main building's architectural volumes according to the speculations summarized in CAD files and Vitruvian most general rules (Cesariano, 1521). This peculiar stage was extremely important in the understanding of Sangallo's work on the Mausoleum, since it represented the first step in considering the third dimension in the whole design process. The plan and the section view, even if the latter shows a simplified perspective in Giuliano's sketch, were somehow related but some connections, just like the circular stairways, were apparently not considering proper space in terms of height and accessibility. The different digital models were prepared considering these premises, proposing some solutions whose feasibility was tested according to different slopes of the roof, different wall thickness at the floors, height and depth of staircases and so forth, following some principles already expressed in scientific literature (Apollonio, Gaiani & Sun, 2012). According to recent studies (Frommel, 2014) the monumental building could have been placed in the south side of the constantinian Saint Peter church, close to a couple of already existing circular buildings, the Saint Andrew and Saint Petronilla churches. The 3D model was initially sketched according to the linear and circular space possibly located in this specific area; the massive Saint Peter church could also have influenced light coming into the mausoleum, as later explained. Comparing the Mausoleum diameter with that of the mentioned *rotundas*, which can be inferred from the reconstruction of the ancient Basilica of San Pietro plan by Tiberio Alfrano (1590), nine meters in discrepancy can be measured if the mausoleum is compared to the eastside building and thirteen for the westside one (Figure 9).

The basic parametric module was extensively used in the reconstruction, beginning from the plan view, and copying it several times in sequence, to identify the total width and radius of the circles traced by the Giuliano to precisely sketching perimetral walls. On the inner ground floor were clearly identified whole circular columns and three-quarter columns leaning on staircases walls, while on the outside semi-columns were placed against the perimeter walls; thus, three other circles trace the position of the steps that make up the podium on which the monumental building stands. Tracing the diagonals with a 45° angle towards vestibules, starting from the position of the niches, the symmetry axes of the chapels are clearly visible (Figure 10). Rotating the central axis passing through the niches and vestibules with a 15° angle, the position of the three-quarter columns center was identified. The chapels are shielded by two columns positioned in relation to the central axis at a distance of ¾ *braccio fiorentino*. Then, the width of the stairwells, the vestibules and the niches were defined using the *braccio fiorentino* again. The same procedure was applied to define the remaining areas identified by the plan view, considering that three different column diameters were identified. The section view, again, seems to depend on the *braccio fiorentino*. Three architectural orders are superimposed following this ratio, even for smaller details (Figure 11).

According to the plan, the section view reveals a 20 meters high building, excluding the pedimental stairs and the top lantern. Following the same criterion, the thicknesses of the walls, the height and width of all the aisles and the windows (whose dimensions were probably set by Giuliano using some rectangular traces close to the section) were replicated. A slight pencil mark, probably sketched by Sangallo, leads to a roof section in which the third order could have also been aligned to the two underlying ones, in a perfectly cylindrical monument. Both hypothesis, the full cylindrical building and the tapered one, were modelled (Figure 12).

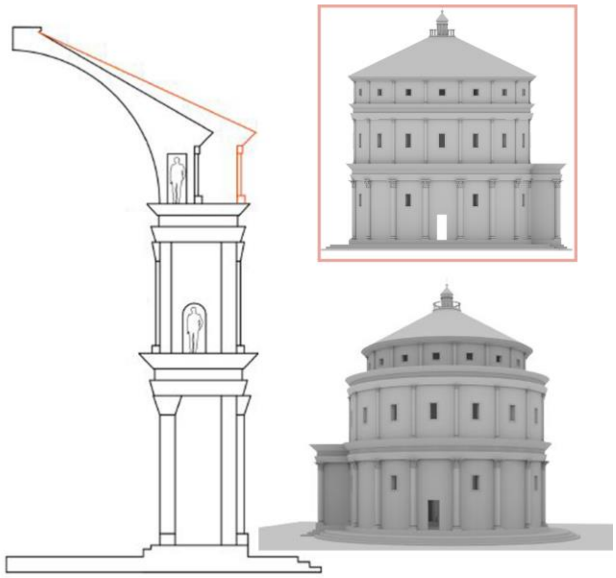

Fig. 12: The roof top hypothesis inferred by different lines traced on the original drawing by Giuliano: the cylindrical building was modeled following both top solutions but only the tapered one was selected as the most proportioned.

An initial and paramount discussion was brought by the question on which one of the section or plan views was sketched before. Probably the section view was designed even before the plan and, anyway, the last one was deeply influenced by the elevation, following the first sketch. If the section view is carefully observed, the sarcophagus is clearly hosted by a vestibule aligned to the circular profile of the circular building. In the plan view, on the other hand, the same vestibule has been enlarged with a projecting space that modifies, at least on the ground floor, the circular orientation of the perimeter wall, according with the elevation. Some geometric hypothesis were developed, leading to a vestibule wider than the others to host the Julius II's tomb, perfectly in front of the main entrance. These arguments reinforce the thesis that the plan view has been authored after the section sketch.

Since the lantern is incomplete in the original drawings, a roof top similar to the one in Santa Maria delle Carceri was developed; also a repetitive scheme for windows was proposed. They were placed according to a radial layout.

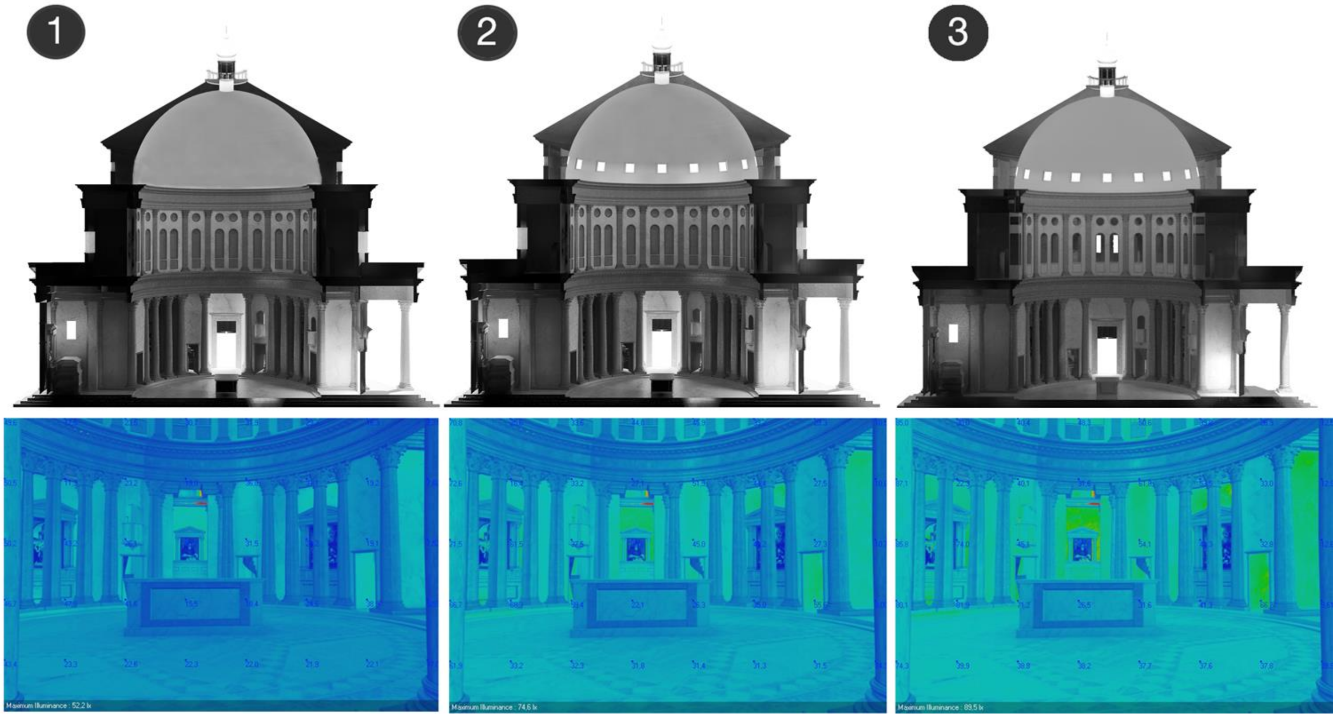

Fig. 13: Light analysis for three different hypothesis in windows layout: light getting into from the lantern (average 52,2 lux), from lantern and radial windows along the dome (average 74,6 lux) and from lantern, dome and open niches (average 89,5 lux).

The interior was modeled following mostly the plan: the original elevation view shows very little detail for decorations, mouldings and *fascias*. The trabeation, for example, is poorly represented, according to the representation scale. However, it is divided into three parts following very precise proportions, similar to the architectural interior detail in Santa Maria delle Carceri, which was replicated in the digital reconstruction. While the upper part of the internal room follows this sample, the lower decoration and many simple pediments were replicated following the original sketches by Giuliano, collected in the Taccuino Senese.

5. The impact of light in reconstructing the architectural space

To better explore the speculated design intent for this building, some photorealistic views were produced to combine the morphology with the visual look for architectural details and sacred space. Also materials were considered to reach successfully this goal, especially when referred to their behaviors toward natural light, as this will be introduced later in this chapter. A wide use of marble was supposed for interiors and exteriors according to actually erected coeval buildings in Rome and the Sangallo's legacy replicated from his other works. In order to achieve a more accurate representation of surfaces during rendering of the reconstructed building, travertine and limestone with harmonized and light differences in color were chosen, spanning from a variable 1.480 to 1.690 IOR value. When light passes through a transparent surface, in fact, the light is typically bent or distorted. This distortion is known as *refraction* and the amount of refraction is known as the *index of refraction* (IOR).

Materials were set up in order to apply a well consolidated *BRDF function*, widespread in many commercial rendering software, and get a realistic visualization of the modelled architecture. The *bidirectional reflectance distribution function* (BRDF) is a function of four real variables that defines how light is reflected at an opaque surface (Nicodemus, 1965) and it is widely employed in computer graphics to replicate the optics of real-world light. The algorithm was again implemented in *Autodesk 3D Studio Max 2016*. Taking advantage of photometric values for a natural light source as embedded into the software, orientation and windows layout were carefully tested, in order to find a proper combination with materials. Since illuminance is the luminous flux incident on a surface of unit area and it measures how much

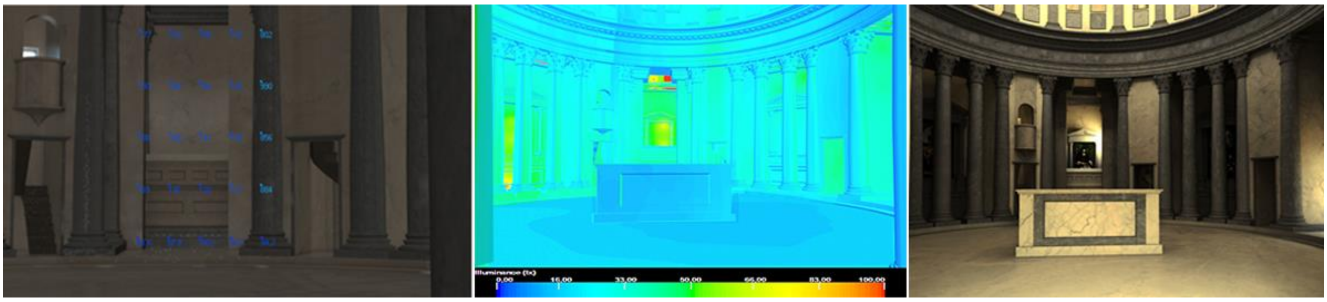

Fig. 14: Light analysis and final realistic renderings showing how the niche, dedicated to the Pope's sarcophagus, should have been the most illuminated internal area.

energy has fallen on a surface, a systematic and recursive study was carried out on different reconstruction hypothesis. Illuminance, whose dimensional unit is the *lux*, is a function of the distance from the light source and it expresses how materials and shapes affect the perception of spaces. That is why it is considered important in architectural simulations.

Three different solutions were tested: an accurate reconstruction following the original drawings, with a closed dome, a solution where many radial windows were placed along the dome to increase internal illuminance and a third hypothesis where also niches at the first floor are open (Figure 13).

Using the program tool that punctually detects the illumination values on a surface through the construction of a grid, it was possible to detect the sunlight values, expressed in lux, which cover the floor surface and the area where the sarcophagus is located.

Evidently, the solar beams that mostly hit the floor come from the lantern and the dome, reinforced by the diffused light that comes from the gallery created at the second level.

When all the upper niches are kept blind there are almost totally dark points in the inside of the building while, opening the niches, a maximum 429 lux value is measured on the floor surface, also due to the upper lantern.

The simulations performed shows that the niche housing the sarcophagus is the most illuminated area of the mausoleum, not the main altar (Figure 14). Due to the two openings on the side walls of the forepart the sarcophagus is illuminated by a very intense light compared to the other parts of the monument with an average value of 100 lux at 1.00 pm of a summer day.

This results reinforce the thesis that the avant-corps could actually be a place destined to immediately capture the view of visitors, therefore the most suitable place to host the Pope to whom the monument is dedicated.

6. *Conclusion*

Even if this work could be considered as a work in progress research, due to the several difficulties arising from the evaluation of often fragmented data and the interpretation of elements not always clearly identified in original drawings, it introduces a possible new perspective in the deep understanding of Giuliano da Sangallo's approach towards the architecture and its representation.

The wide use of digital models, authored following strict proportional and numerical rules related to drawings, was adopted in two one-year long master thesis activities, where many different hypothesis on the Mausoleum reconstruction were tested.[1]

The final developed proposal is primarily a formal reconstruction based on sketches and traditional building details inferred by paintings and sculptures dated back to the same historic period. It is a collection of information focused on how the building could have been erected, following formal rules and constructive solutions,

[1] The digital reconstruction of the Sangallo's architectural proposal introduced in this paper, probably destined to the Julius II's mausoleum, is a further development of a wider set of studies, whose preliminary and partial outcomes were included in two master thesis defended by Alice Cancilla and Elena Masina at the Alma Mater Studiorum, University of Bologna. This work is far more detailed than what introduced in their thesis, referring to a research activity that persisted and it is still in progress.

from the main structure to the minute proportional garnish. Future perspectives of this research could likely encompass a more detailed interaction of digital building components and their perception into realistic environments, based on already consolidated scientific works on proper semantics developed by Renaissance drawings (Gaiani, 2019), in order to simulate a real building construction taking into account time needed by different stages and resources to be committed for the actual construction. While the geometry represented by the shape of all the assembled objects is clearly identified in a digital model, in fact, information attributes from an extended database connected to the digital representation could lead to a better understanding of the many facets of the inner Renaissance design intent by Giuliano da Sangallo.

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